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A LEAST-SQUARES PROCEDURE FOR DIRECT  
ESTIMATION OF AZIMUTH AND VELOCITY OF  
A PROPAGATING WAVE

E. A. Flinn, et al

Teledyne Geotech

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# A LEAST-SQUARES PROCEDURE FOR DIRECT ESTIMATION OF AZIMUTH AND VELOCITY OF A PROPAGATING WAVE

BY

E.A. FLINN and D.W. McCOWAN

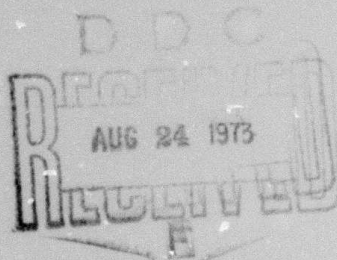
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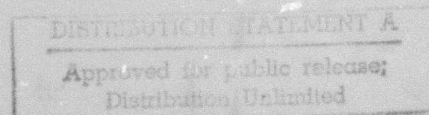
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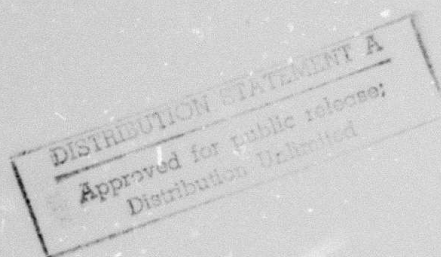
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A LEAST-SQUARES PROCEDURE FOR  
DIRECT ESTIMATION OF AZIMUTH AND VELOCITY  
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## ABSTRACT

A least-squares technique for the estimation of the phase velocity and apparent azimuth of a plane wave propagating across a horizontal array is developed and applied to two events: the Rayleigh waves from an Alaskan earthquake recorded on the seven-element UBO long-period seismic array, and the acoustic-gravity waves from a presumed atmospheric nuclear explosion, recorded on part of the thirteen-element microbarographic array at LAMA. Both time-domain and frequency-domain formulations of the technique were used, i.e., we used both the multichannel correlation and spectral matrices to measure the interchannel time shifts. The results suggest that the time-domain application may provide a useful algorithm for signal detection. The phase velocity dispersion results in both cases agree reasonably well with theory and with group velocities calculated by another method. Prewhitening (flattening the spectral peaks by linear filtering) was tested as a means of preventing spectral leakage from contaminating the derived phase velocity and azimuth; the results suggest that prewhitening decreases the standard error when the signal-to-noise ratio is high, but is ineffective when the signal-to-noise ratio is low.

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## INTRODUCTION

McCowan and Flinn (1968) described a least-squares method for estimating the  $N-1$  interchannel time shifts between corresponding phases of a plane wavefront crossing a horizontal array of  $N$  elements. The technique made use of estimates of the interchannel time shifts based on the time lags between all possible pairs of elements as measured from the multichannel cross-correlation matrix. It was shown that this led to  $N/2$  degrees of freedom for estimation of each of the time shifts.

In this paper we discuss a simple extension of this method: the direct least-squares estimation of the propagation velocity and apparent arrival azimuth of the hypothetical propagating wavefront. Since in this case there are only two variables to solve for (instead of  $N-1$ ), the number of degrees of freedom is increased to  $(N^2-N-8)/4$ , and thus we would expect greater stability in the estimates, provided of course that what we are observing is indeed a plane wave).

Another motivation for this extension of the method is detection analysis. At least one detection algorithm now in use (the  $N-4$  multichannel correlator; see Brown, 1962) directly measures the signal phase velocity and

azimuth of arrival. The detection process includes measurement of the persistency in time of the propagation parameters. The least-squares method presented here has similar characteristics, but in addition to the estimation of the propagation parameters themselves, the present technique gives estimates of the standard errors of the derived variables. We might therefore expect that these standard errors, displayed as a function of time, may also be a practical detection algorithm, since presumably they should be significantly smaller when a signal is present than when the records contain random and incoherent noise.

In this paper we describe and demonstrate this method of estimating the propagation parameters and their errors. A more detailed discussion and additional results are given by McCowan and Flinn (1970).

## THEORY

When a plane wavefront crosses a horizontal array of  $N$  elements, as illustrated in Figure 1, the time interval between the arrival of a corresponding wave feature at two different elements is:

$$\Delta t_{ij} \equiv (t_j - t_i) = (x_i - x_j) \frac{\sin \theta}{V} + (y_i - y_j) \frac{\cos \theta}{V} \quad (1)$$

Since this equation must hold between all possible pairs of elements, the problem of calculating  $V$  and  $\theta$  from the observed is clearly over-determined. For example, with an array of four elements, the equations are:

$$\begin{bmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \\ x_4 - x_1 & y_4 - y_1 \\ x_3 - x_2 & y_3 - y_2 \\ x_4 - x_2 & y_4 - y_2 \\ x_4 - x_3 & y_4 - y_3 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} t_1 - t_2 \\ t_1 - t_3 \\ t_1 - t_4 \\ t_2 - t_3 \\ t_2 - t_4 \\ t_3 - t_4 \end{bmatrix} \quad (2)$$

where the unknowns are:

$$f_1 = \frac{\sin \theta}{V}, \quad f_2 = \frac{\cos \theta}{V} \quad (3)$$

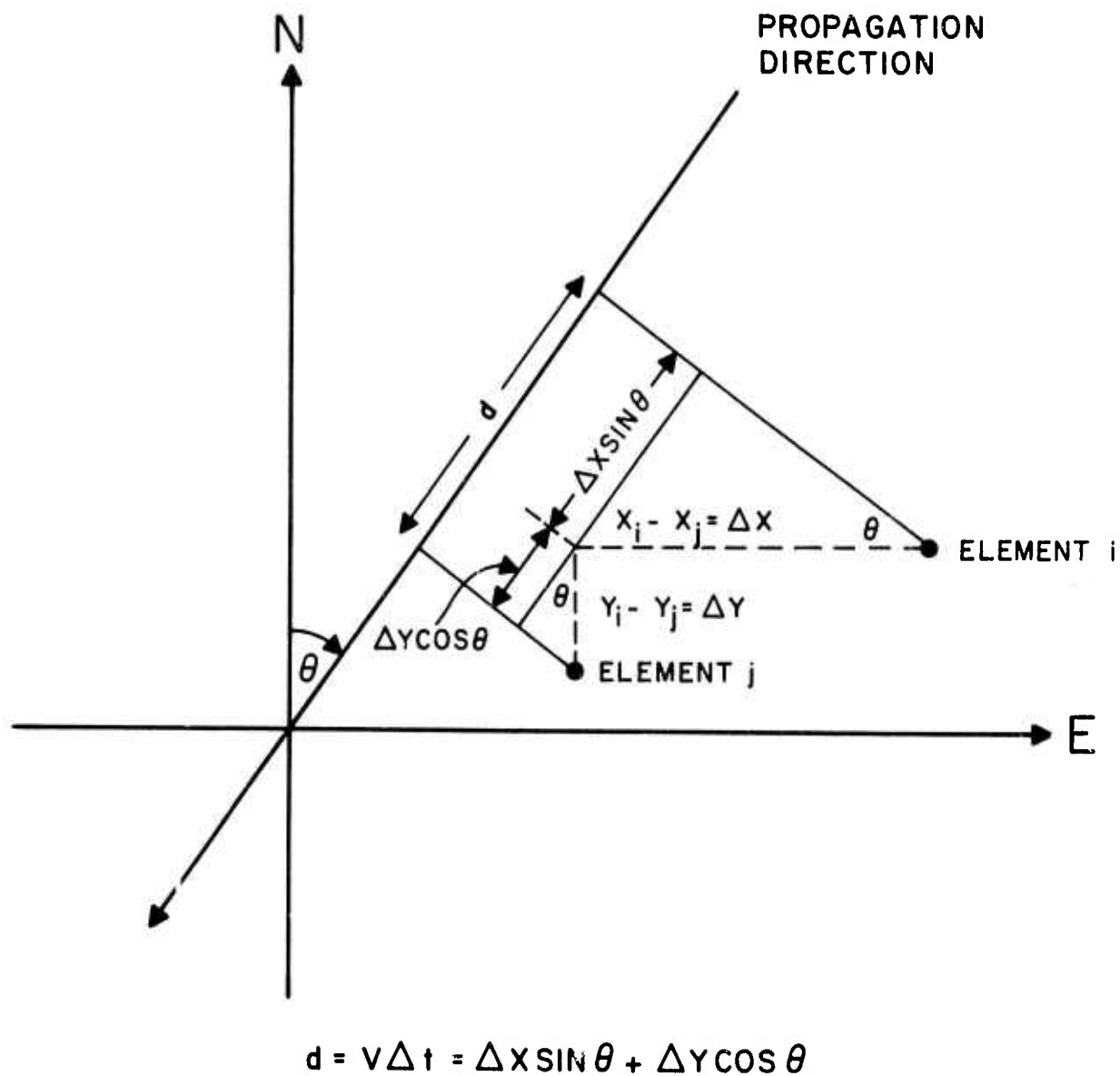


Figure 1. Notation and sign conventions for a plane wave propagating across an array.

There are  $(N^2-N)/2$  equations when there are  $N$  array elements, so there are  $(N^2-N-8)/4$  degrees of freedom left over to provide information about the errors in the estimates  $\hat{f}_1$  and  $\hat{f}_2$  (Anderson, 1958).

In matrix form, equation (2) is:

$$H\tilde{f} = \tilde{t} \quad (4)$$

and the least-squares estimate of the unknown vector is:

$$\hat{\tilde{f}} = (H'H)^{-1}H'\tilde{t} \quad (5)$$

Providing the errors are small, reasonable estimates of the propagation phase velocity and azimuth can be obtained from the relations:

$$\hat{V} = (\hat{f}_1^2 + \hat{f}_2^2)^{-1/2} \quad \theta = \tan^{-1}(\hat{f}_1/\hat{f}_2) \quad (6)$$

Again from least-squares theory, the variances and covariance in the original unknowns  $\hat{f}_1$  and  $\hat{f}_2$  are:

$$\sigma_1^2 = \frac{1}{D} [\tilde{t}'\tilde{t} - \hat{\tilde{f}}'(H'H)\hat{\tilde{f}}][(H'H)^{-1}]_{1,1} \quad (7)$$

$$\sigma_2^2 = \frac{1}{D} [\tilde{t}'\tilde{t} - \hat{\tilde{f}}'(H'H)\hat{\tilde{f}}][(H'H)^{-1}]_{2,2} \quad (8)$$



and

$$\sigma_{12}^2 = \frac{1}{D} [\hat{f}'_1 (H'H) \hat{f}_2] [(H'H)^{-1}]_{1,2}$$

where

$$D = 1/2(N^2 - N) - 2$$

If the errors are small, a Taylor series expansion of equation (6) about  $V$  and  $\theta$  gives the variances in the estimates of  $V$  and  $\theta$ :

$$\sigma_V^2 \approx \left(\frac{\partial V}{\partial f_1}\right)^2 \sigma_1^2 + \left(\frac{\partial V}{\partial f_2}\right)^2 \sigma_2^2 + 2 \left(\frac{\partial V}{\partial f_1}\right) \left(\frac{\partial V}{\partial f_2}\right) \sigma_{12}^2 \quad (10)$$

$$\sigma_\theta^2 \approx \left(\frac{\partial \theta}{\partial f_1}\right)^2 \sigma_1^2 + \left(\frac{\partial \theta}{\partial f_2}\right)^2 \sigma_2^2 + 2 \left(\frac{\partial \theta}{\partial f_1}\right) \left(\frac{\partial \theta}{\partial f_2}\right) \sigma_{12}^2 \quad (11)$$

Substituting partial derivatives calculated from (6), equations (10) and (11) become:

$$\sigma_V^2 \approx \hat{V}^6 [\hat{f}_1^2 \sigma_1^2 + \hat{f}_2^2 \sigma_2^2 + 2\hat{f}_1 \hat{f}_2 \sigma_{12}^2] \quad (12)$$

$$\sigma_\theta^2 \approx V^4 [\hat{f}_2^2 \sigma_1^2 + \hat{f}_1^2 \sigma_2^2 - 2\hat{f}_1 \hat{f}_2 \sigma_{12}^2] \quad (13)$$

Thus equations (6) and (12) - (13) constitute a least-squares method for obtaining estimates of the phase velocity and arrival azimuth of a propagating waveform, together with their respective variances, provided we can reliably measure the  $(N^2/N)/L$  interelement time delays. If the errors are small, the linear Taylor series expansion is valid out to several standard deviations. Thus if  $t^2$  has a normal distribution, so do  $V$  and  $\theta$ , approximately.

There are two direct ways to measure these delays. The first is to compute the cross-correlation matrix and measure the delays by finding the time lag where each cross-correlation function is a maximum. This method has the advantage that the averaging implicit in computing the correlation functions smoothes out the effect of uncorrelated noise. Measuring the delays in this way can be expected to give values of phase velocity and arrival azimuth which are averages over all frequencies.

The other way to measure the delays is to compute them from the cross-spectral matrix

$$S_{ij}(\omega) = E[X_i^*(\omega)X_j(\omega)] \quad (14)$$

where  $E[\cdot]$  denotes the expected value and the asterisk indicates complex conjugation. The spectrum of a propagating signal is:

$$S_{ij}(\omega) = A(\omega) \exp[i\omega\tau_{ij}(\omega)] \quad (15)$$

where  $\tau_{ij}$  is the time delay between array elements  $i$  and  $j$ . One difficulty is the necessity to make sure that the phase spectrum for each cross-spectrum is properly "unwound", i.e., the periodicity of phase removed. This can lead to trouble when two elements of the array are far apart. If difficulties arise for this reason, however, then the time-domain calculation is also ambiguous, because the cross-correlation functions will be longer and there will be an ambiguity of peak correlation. The major advantage of the frequency-domain approach is that independent information is obtained at each frequency, provided of course that the cross-spectral estimates are themselves uncorrelated between frequencies.

To summarize, if the errors in  $\hat{t}$  are small and normally distributed, we expect that the estimates of phase velocity and azimuth of the propagating wavefront to be approximately normally distributed variables.

Therefore the standard deviations given in equations (12) and (13) should represent familiar confidence intervals.

The velocity dispersion that is measured by the least-squares technique is an array-dependent phenomenon. Neither spectral matrices nor correlation matrices contain any information about path-dependent dispersion: they are both sensitive only to phase differences, not absolute phase. Therefore the dispersion results obtained by the technique described here depend only on structure of the propagation medium in the vicinity of the array. This is advantageous in that a localized measurement of structure can be made, but the advantage may be offset by the problem of resolution: small arrays, though yielding consistent results, see only a small portion of the wave propagation process. Thus we must weigh the results of the technique against the limited scope of observation inherent in small arrays.

Smart (1971) has shown that leakage of power between frequencies can cause misinterpretation of frequency-wavenumber spectra. That is, an apparent energy peak on the usual display of power contoured as a function of two wavenumber components may not represent a parcel of energy travelling with phase velocity equal to the wavenumber location of the peak divided by the frequency

of the cross-section; the peak may simply be leakage along lines of constant wavenumber from a true spectral peak at some other frequency. Smart has proposed a method for eliminating this effect.

We were not sure that this leakage might not cause errors in the values of phase velocity and azimuth derived by the method presented in this report, since the spectra of the Rayleigh and acoustic-gravity waves do contain large peaks. To test the leakage effect we performed all analyses twice, once with the original records, and once after "prewhitening", i.e., linear filtering to lower the spectral peaks (McCowan and Flinn (1970) for details).



## PROCEDURE

Two events were selected to demonstrate the method: the Rayleigh waves from an Alaska earthquake recorded in Utah by the Unita Basin Observatory array of long-period vertical seismometers, and an acoustic-gravity wave signal from a presumed atmospheric nuclear explosion, recorded in Montana by part of the Large Aperture Microbarograph Array (Cook and Bedard, 1971). These two events were chosen because of easy availability of digital data, and good signal-to-noise ratio.

Information about these events was taken from the NOAA/NOS Preliminary Determination of Epicenters releases; a summary is given in Table I. Relative locations of the array elements can be obtained from Carpenter (1965). The records used are shown in Figures 2 and 3, along with the array beams directed toward the NOS epicenters. LAMA elements D1 - D4 were not used in this study.

The responses of the two arrays are roughly circular, and the diameter of the 1 dB contour of both is about 0.09 cycles per kilometer.

The data were detrended and the microbarograph records were bandpass-filtered and decimated by a factor of four. For the leakage effect experiment, the data channels were prewhitened, each independently.

TABLE 1

Event Data\*

Rayleigh Waves Alaskan Earthquake		Acoustic Gravity Waves	
Array	UBO-LPZ		LAMA
Number of Elements	7		8
Sampling rate	1.0 samples/sec		0.5 samples/sec
Time window	10:2:32-10:36:58 GMT		2:20:42-3:28:55 GMT
Group velocity window	3.6-1.1 km/sec		0.313-0.271 km/sec
Date	2 June 1969		8 September 1968
Origin time	9:47:59 GMT		18:59:59 GMT
Latitude	59.5 N		22S
Longitude	144.7 W		139W
Seismic Magnitude ( $m_b$ )	4.7		4.7
Epicentral distance	3181 km		8270 km
Azimuth of source from station (east to north)	322°		212°

\*Based on information taken from NOAA/NOS  
Preliminary Determination of Epicenter Releases

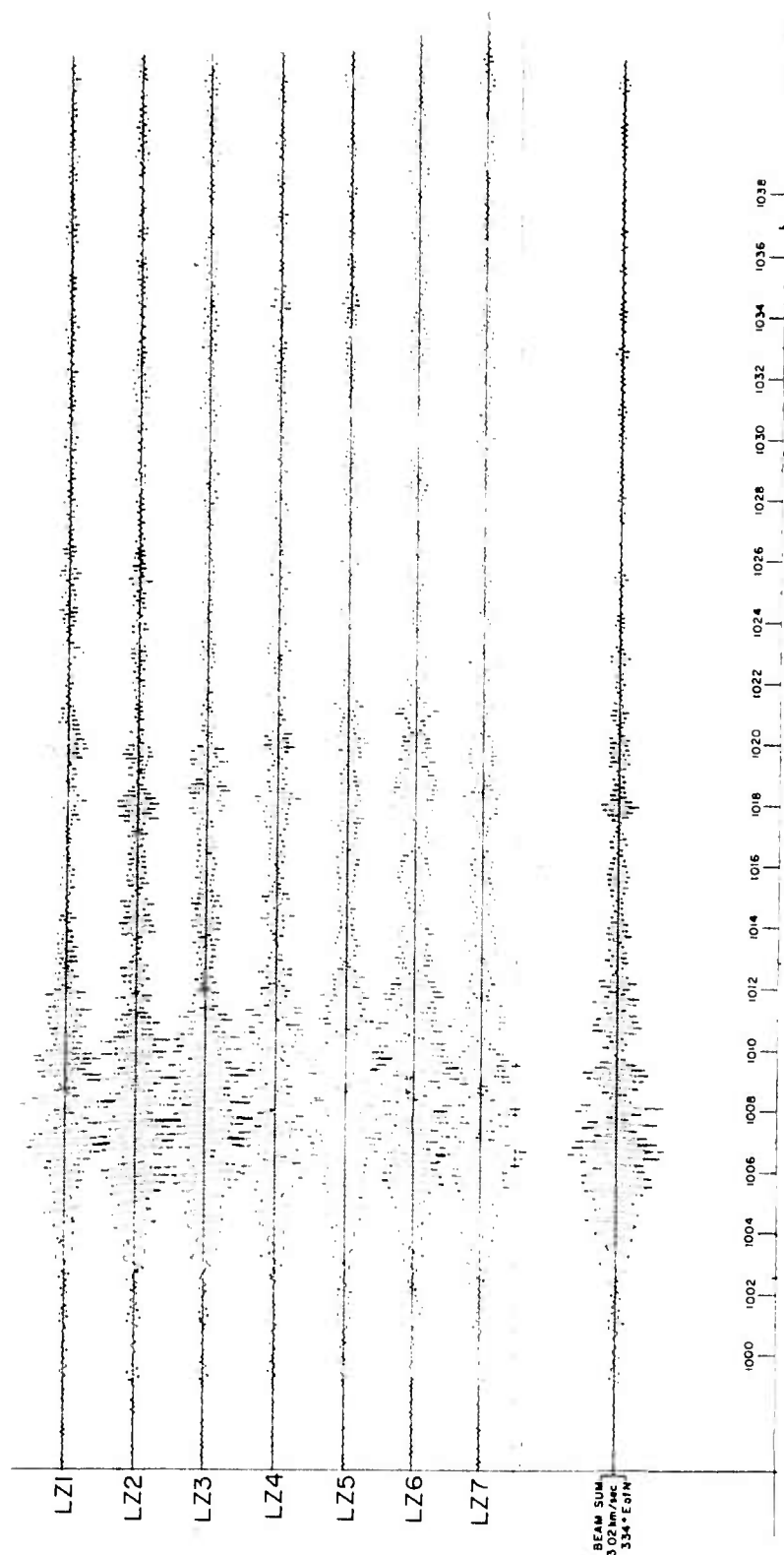


Figure 2. UBO recording of Rayleigh waves from the Alaskan earthquake of 2 June 1969 (see Table 1).

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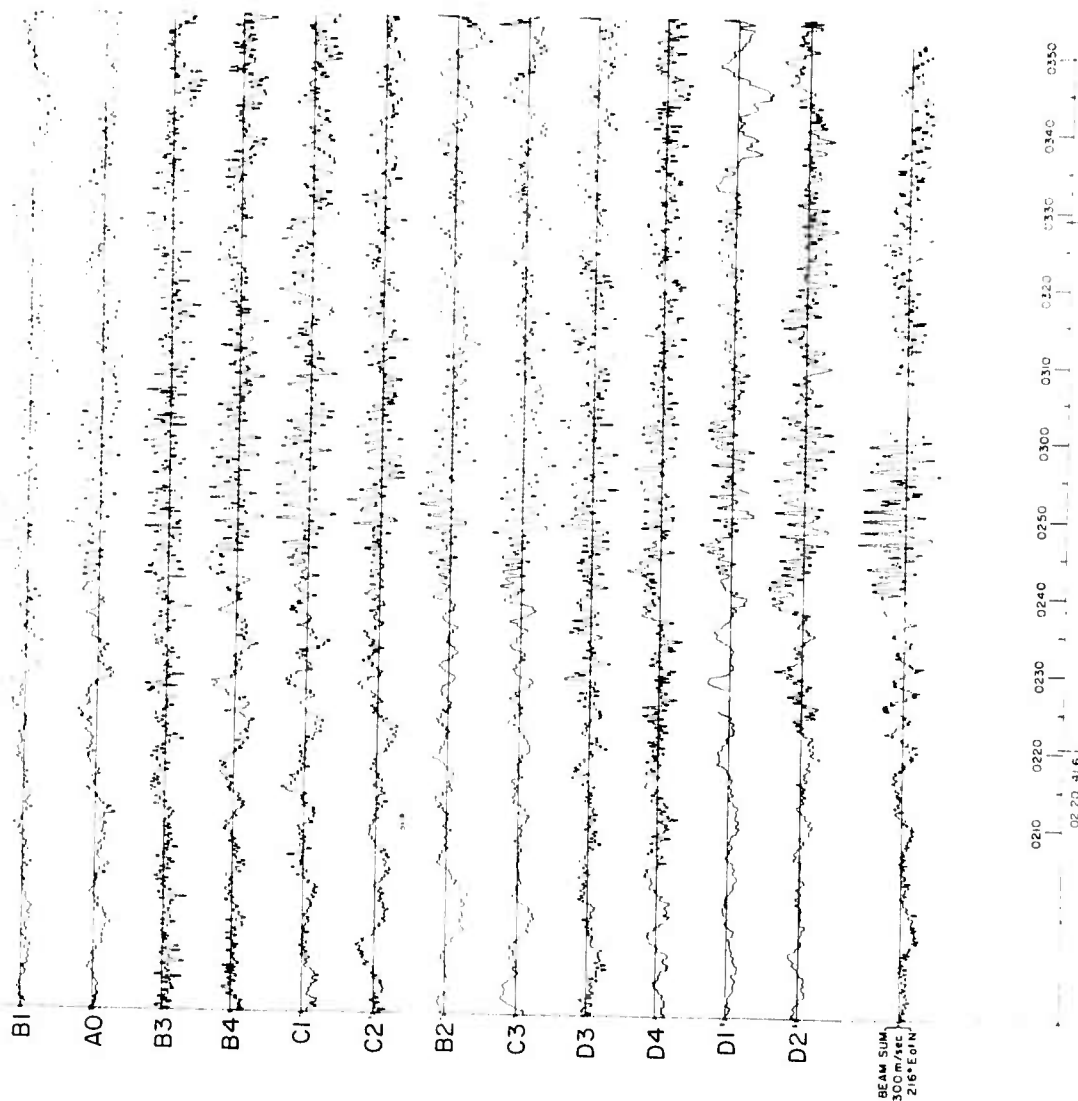


Figure 3. LAMA recording of acoustic-gravity waves from a presumed atmospheric explosion (see Table 1).

For both events the prewhitening filter was a fifty-point least-squares approximate inverse filter, applied in the time domain before computing correlations or spectra (for background, see for example Treitel and Robinson, 1966). The cross-correlation matrices were computed using an algorithm described by Flinn (1968), and the cross-spectral matrices using a Fourier transform method described by McCowan (1968).

We estimated group velocity by two methods. First, we used a process described by Archambeau and Flinn (1965) in which the records are put through a comb of very narrow recursive bandpass filters and the group velocity calculated from the arrival time of maximum envelope amplitude on the filter outputs. The program compensates for instrument group delay, although for the epicentral distances involved here the effect of instrument group delay is negligible.

The second method used to determine group velocity was to compute power spectra of overlapped time windows and to display the results as a function of frequency and time, i.e., frequency and group velocity. To the best of our knowledge, this technique to determine dispersion of acoustic-gravity or seismic waves



was first described by Pfeffer and Zarichny (1963); the particular program used here was described by Cohen (1969). We used 1024-point windows with an overlap of 75%, with 10% cosine taper applied to each data window. Instrument group delay was not compensated for.

The error bars on all phase velocity and azimuth dispersion curves shown here represent one standard deviation; ninety-five per cent confidence intervals can of course be obtained by doubling these errors.

## RESULTS

The results of this study are presented in Table II and Figures 4 through 10. For both events the time-domain calculation (Table II) shows clearly the effects of prewhitening, which gives worse results for the acoustic-gravity waves and better results for the Rayleigh waves, in terms of standard errors of the estimates. To see why this is so we must look at the results in the frequency domain.

The frequency-domain results for both events show little difference between the unprewhitened and prewhitened cases (the prewhitened results are not given here; they are presented in detail by Flinn and McCowan, 1970). The complexity and variability of the phase velocity and arrival azimuth for the Rayleigh waves from the Alaskan earthquake (Figures 4 and 5) are remarkable. It is obvious that scatter of at least one standard deviation is present in these data, since no reasonable plane-layered earth structure yields a phase velocity dispersion curve given by the mean of the estimates in the anomalous region around 0.06 - 0.07 cps (which shows up in both the phase velocity and azimuth plots). The signal-to-noise ratio (Figure 6) appears to be reasonably high, so noise contamination is probably not the

TABLE 2

Time-Domain Calculation Results

	Acoustic-gravity Waves	Acoustic-gravity Waves (prewhitened)	Rayleigh Waves	Rayleigh Waves (prewhitened)
Number of lags	125	125	50	50
Correlation sampling rate (samples/sec)	0.25	0.25	1.0	1.0
Phase velocity (km/sec)	0.316+0.003	0.270+0.026	4.39+2.15	3.02+0.08
Azimuth of arrival (degrees east of north)	214+1	214+8	295+28	334+1

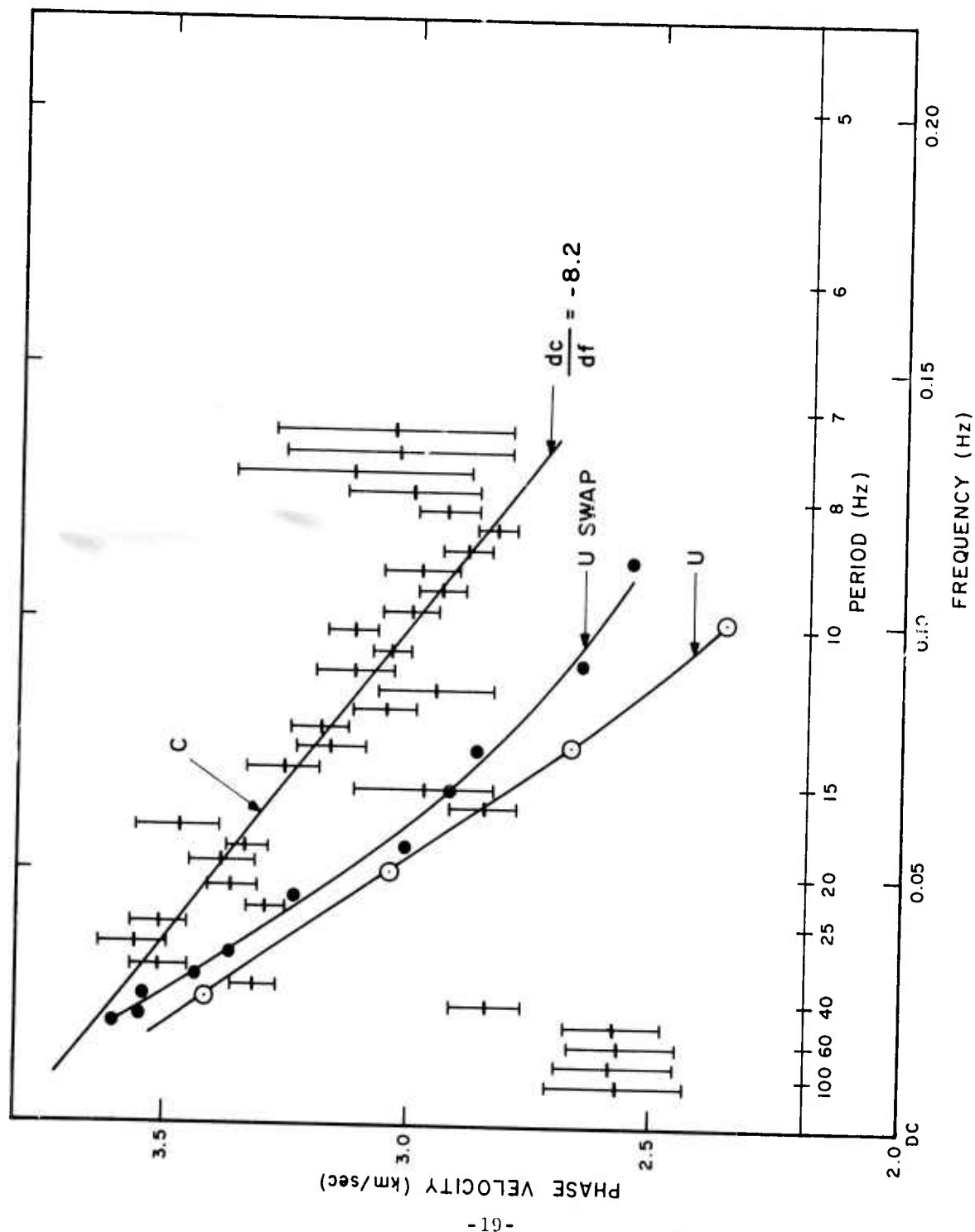
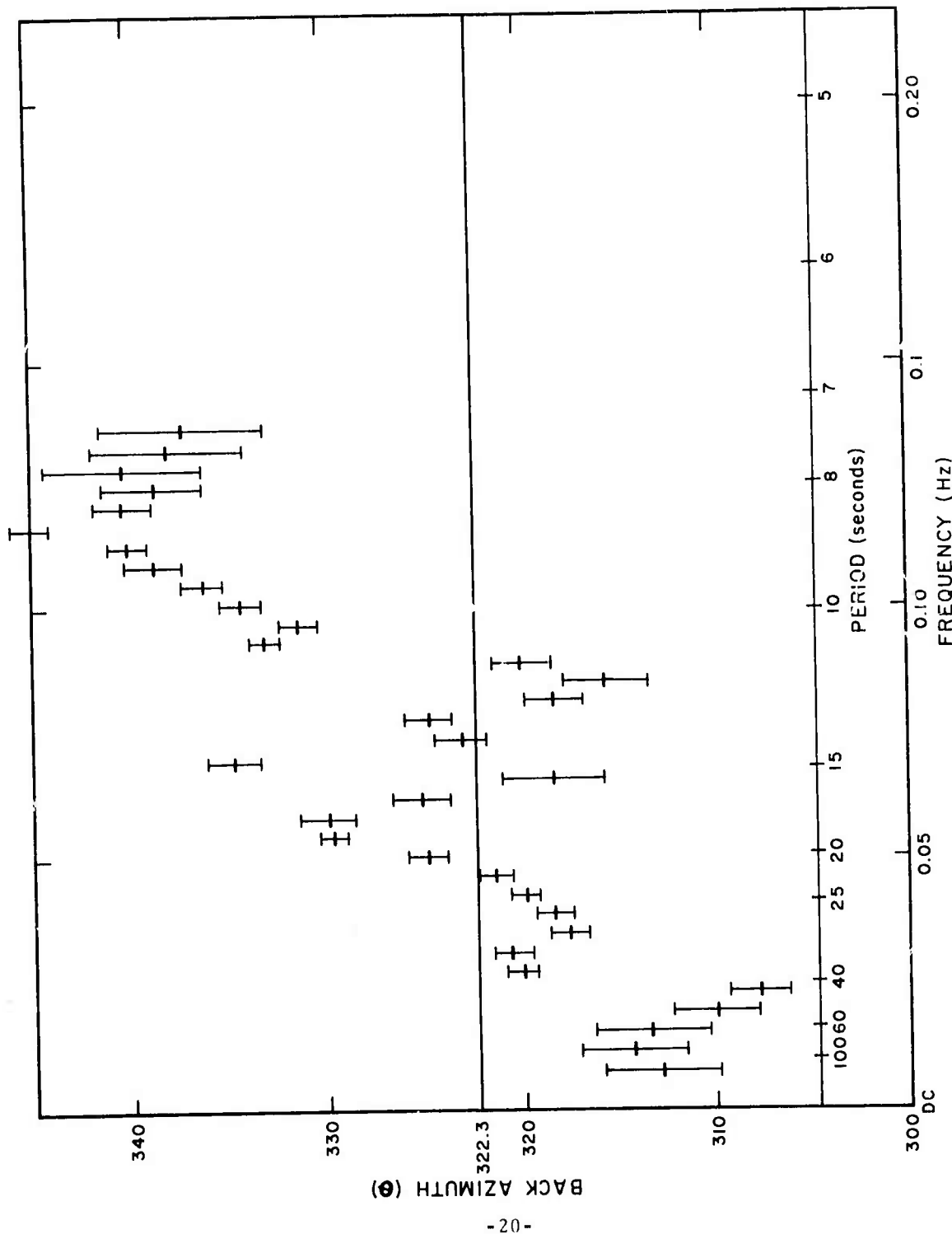


Figure 4. Signal-to-noise ratio for the Alaskan earthquake Rayleigh waves, with an estimate of group velocity added.





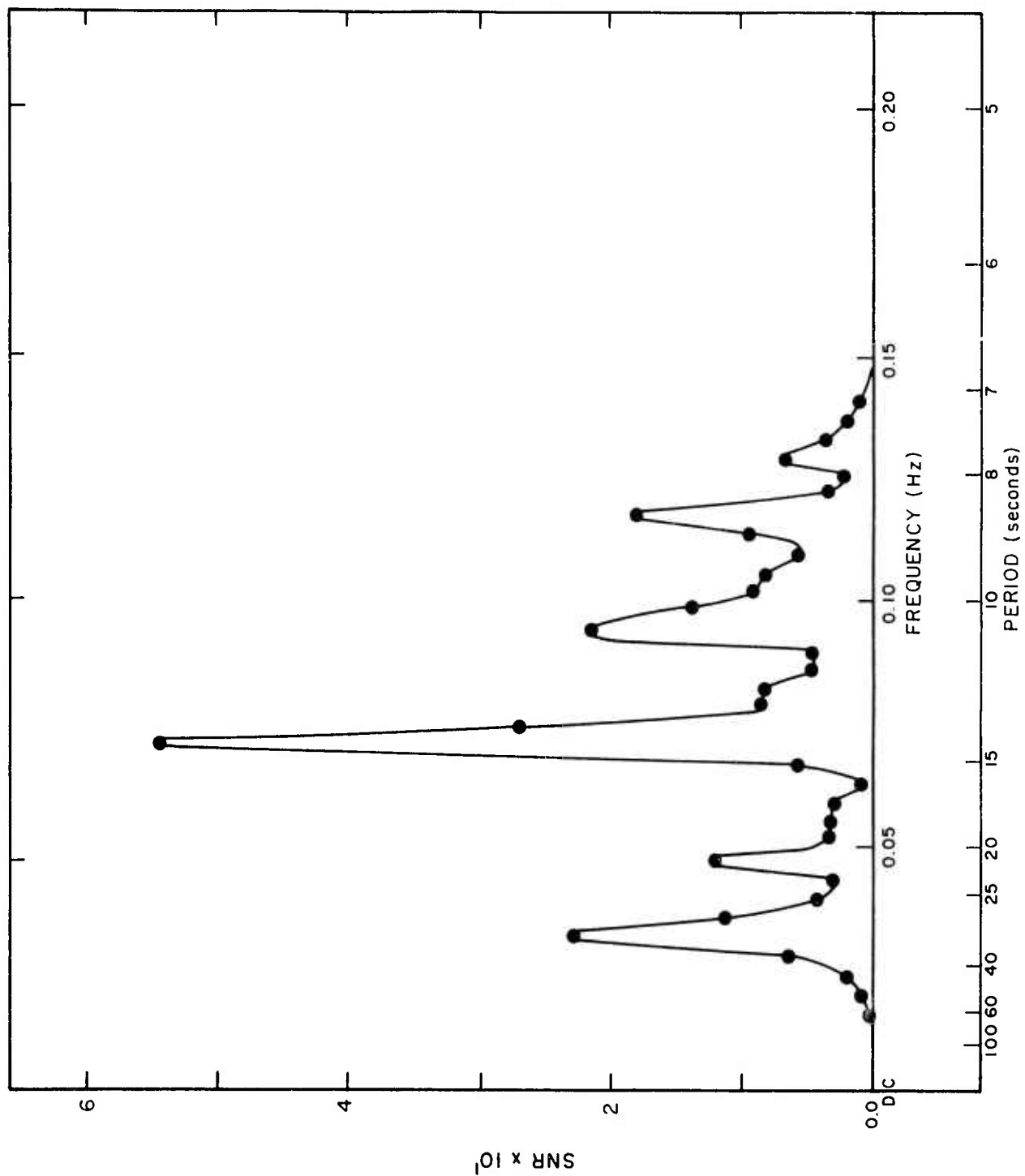


Figure 6. Signal-to-noise ratio for the Alaskan earthquake Rayleigh waves.

cause of the variation in phase velocity (see below for a discussion of the group velocity data shown in Figure 4). It is likely that multipath Rayleigh wave arrivals are present. These might be caused by lateral refractions, the effect of which for dispersed waves is frequency dependent. The difference in arrival azimuth between 8 and 30 seconds period is almost  $30^\circ$ . This can be explained by the complicated travel path from Alaska to UBO, which roughly goes down the west side of the Rocky Mountains and crosses them at a highly oblique angle, which would tend to magnify any refraction effects. Another obvious possible cause is derivation from horizontal layering along the propagation path.

In contrast, the acoustic-gravity waves are comparatively smooth and well-behaved (Figure 7), and the calculated arrival azimuth is practically constant over a broad frequency range (Figure 8). It seems therefore that these records are relatively uncontaminated by winds, which would cause a frequency-dependent refraction similar to that for the Rayleigh waves. The flatness is probably one reason why prewhitening harms these results; another is the comparatively low signal-to-noise ratio of microbarograph

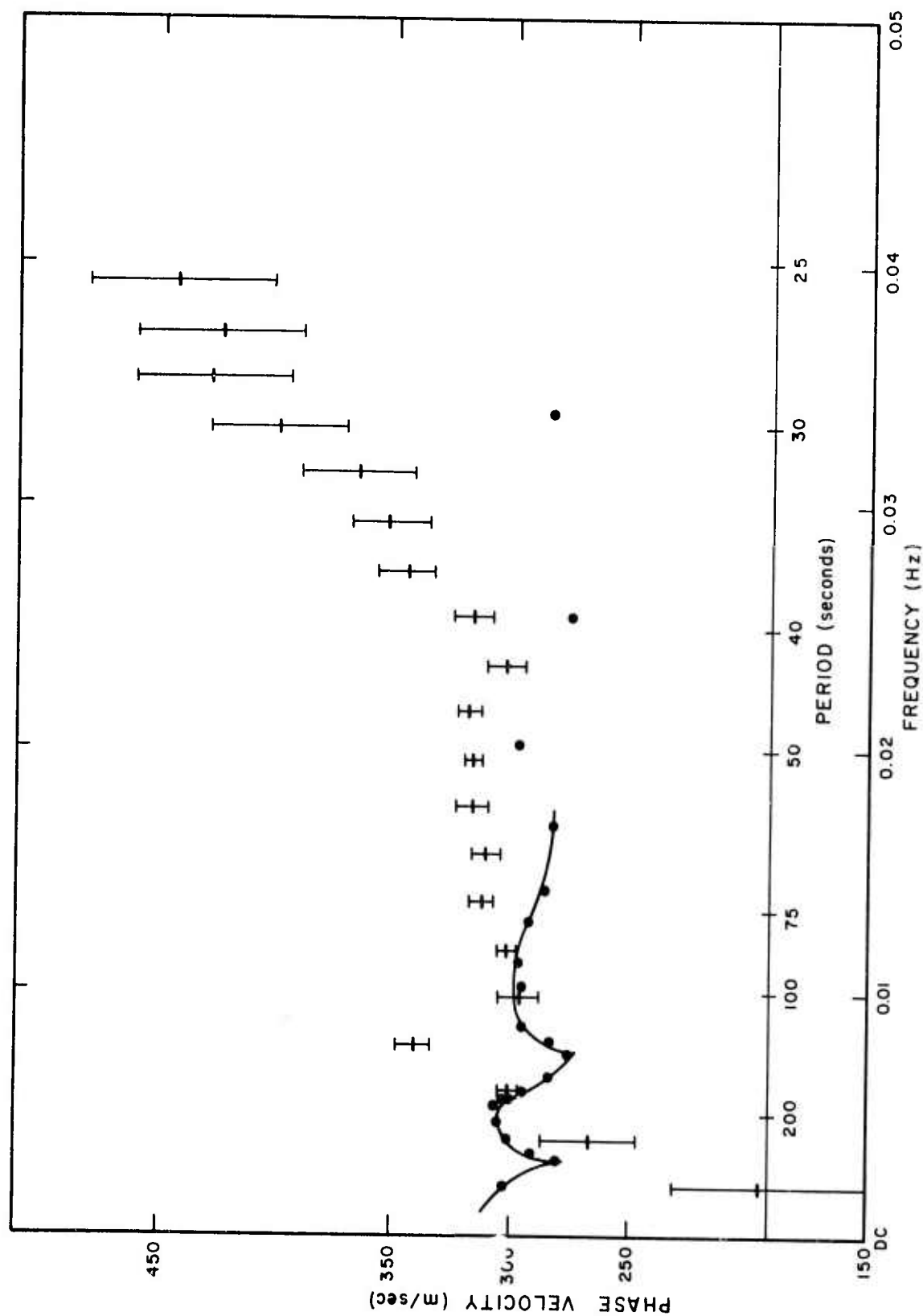


Figure 7. Phase velocity dispersion for the acoustic-gravity waves (no prewhitening).

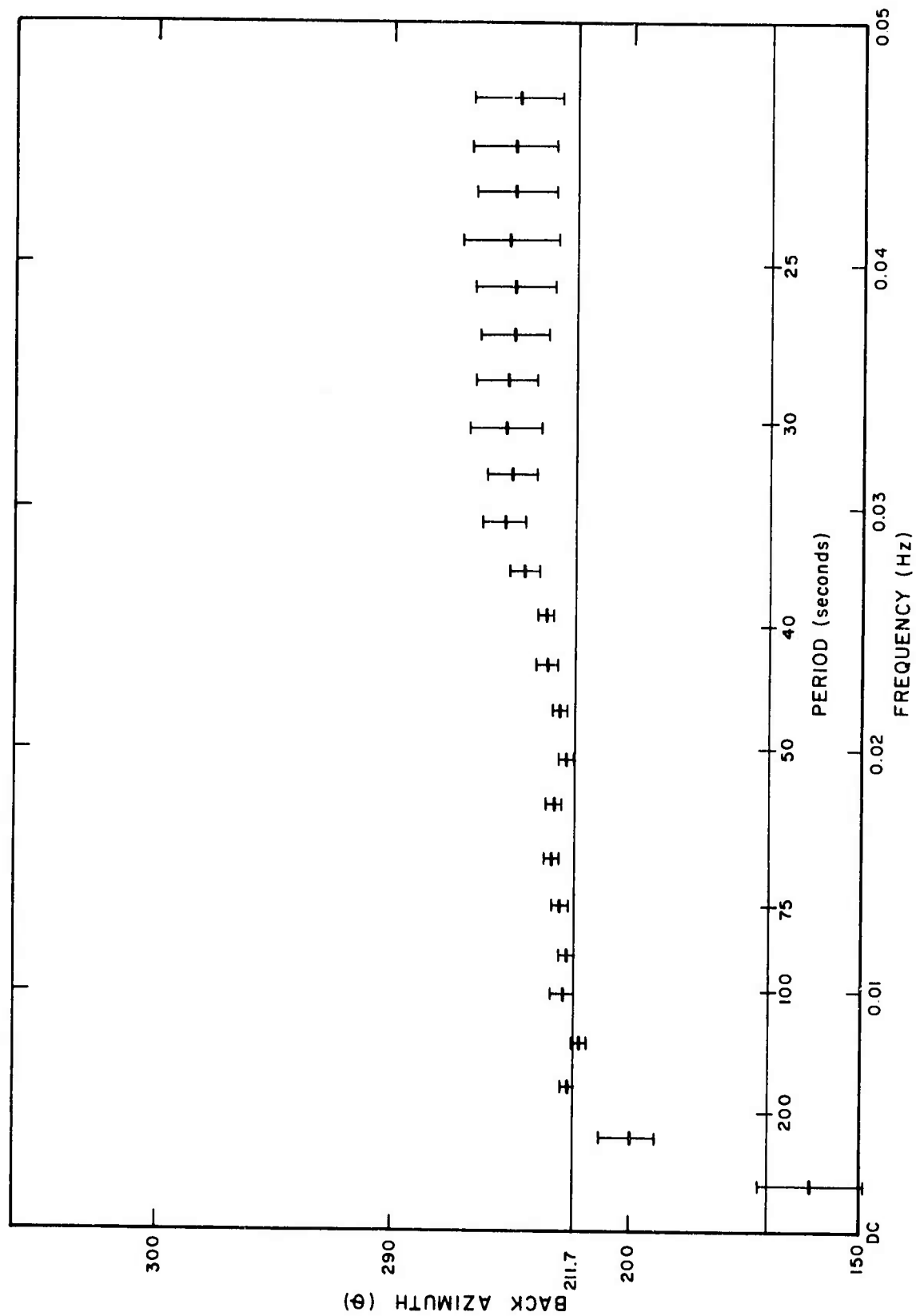


Figure 8. Arrival azimuth dispersion for the acoustic-gravity waves (no prewhitening).

records). We notice from Table II that the arrival azimuths for both the prewhitened and unprewhitened cases are quite close to the great-circle azimuth. We conclude that the addition of white noise to an event of low signal-to-noise ratio should detract from the accuracy of the estimates, and probably cannot enhance them. For the Rayleigh waves, however, prewhitening tends to smooth out in frequency the already high signal-to-noise ratio, thereby giving more uniform results.

In the time domain the Rayleigh wave phase velocity derived from the unprewhitened data is clearly too high, but that derived from the prewhitened data is not only reasonable but agrees with the average over frequency of the frequency-domain calculation.

Figures (4) and (7) also show group velocity dispersion curves calculated by program SWAP (Archambeau and Flinn, 1965). It is interesting to note that the pronounced group velocity minima predicted by Press and Harkrider (1962) are clearly evident in the dispersion curve for the acoustic-gravity waves.

On the basis of the average slope of the phase velocity curve we have also calculated the group velocity dispersion for the Rayleigh waves from the Alaskan earthquake (Figure 4). Not only is the general

shape reasonable, but the actual values agree quite well with the group velocity calculated by SWAP. In these comparisons it must be remembered that program SWAP measures dispersion caused by group delay at the source, along the propagation path, and at the receiver, while the least-squares technique presented here measures only the last effect.

Figures 9 and 10 show time-varying spectra of the data. Group velocities derived by tracing ridge lines (Figure 4) agree reasonably well with those calculated by SWAP except in the vicinity of the 80-second and the 150-second periods for the acoustic-gravity waves, where the SWAP group velocity is about two percent higher than the seismoprint ridge line indicates. This is probably due to mode interference, i.e., to energy interference in the narrow-band filter outputs, to which the SWAP procedure is known to be sensitive (see Archambeau and Flinn, 1965).

Other interesting features appearing on the acoustic-gravity wave seismoprints are discussed by Smart and Flinn (1971).



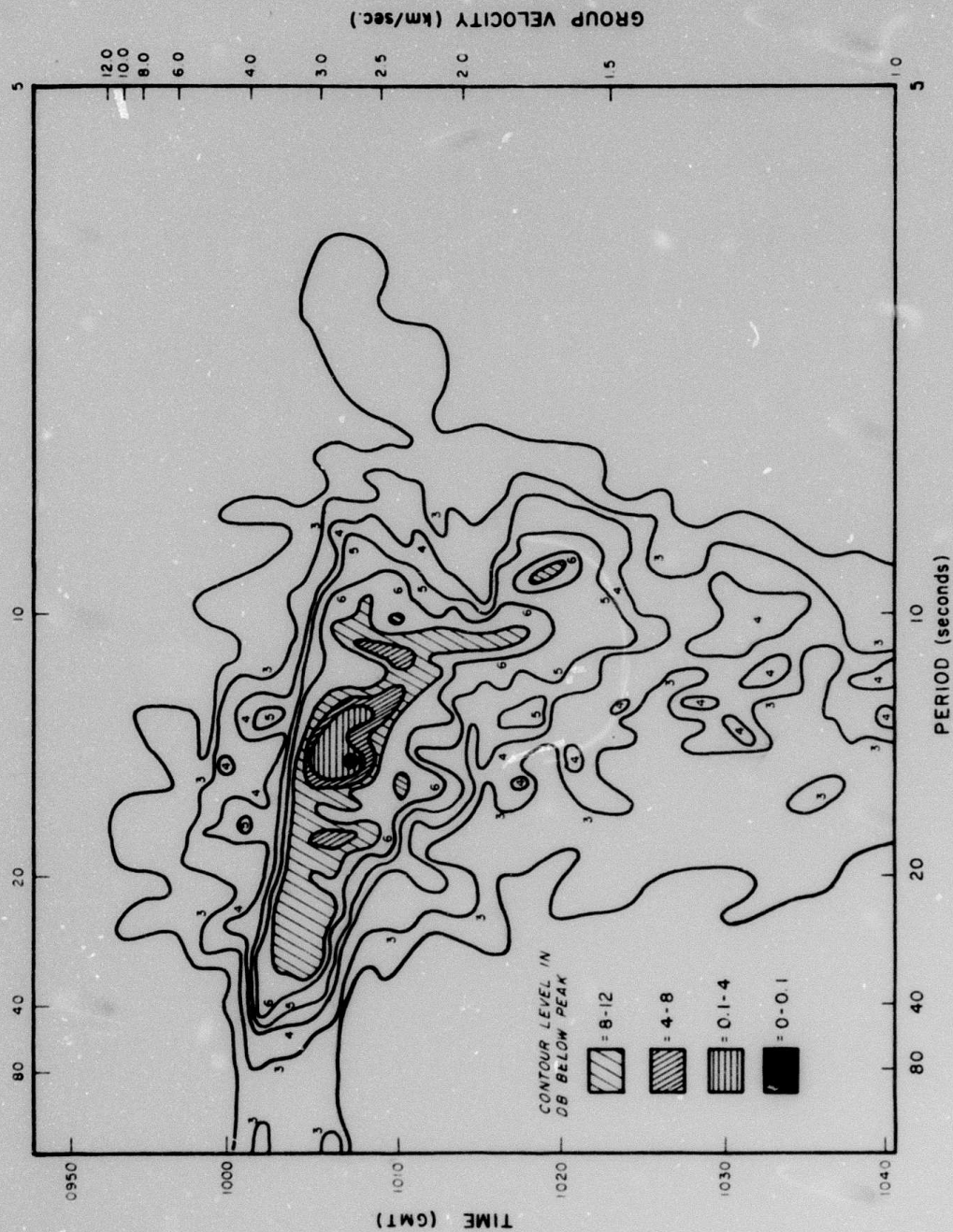


Figure 9. Time-varying spectrum ("seismoprint") of the Alaskan earthquake Rayleigh waves.



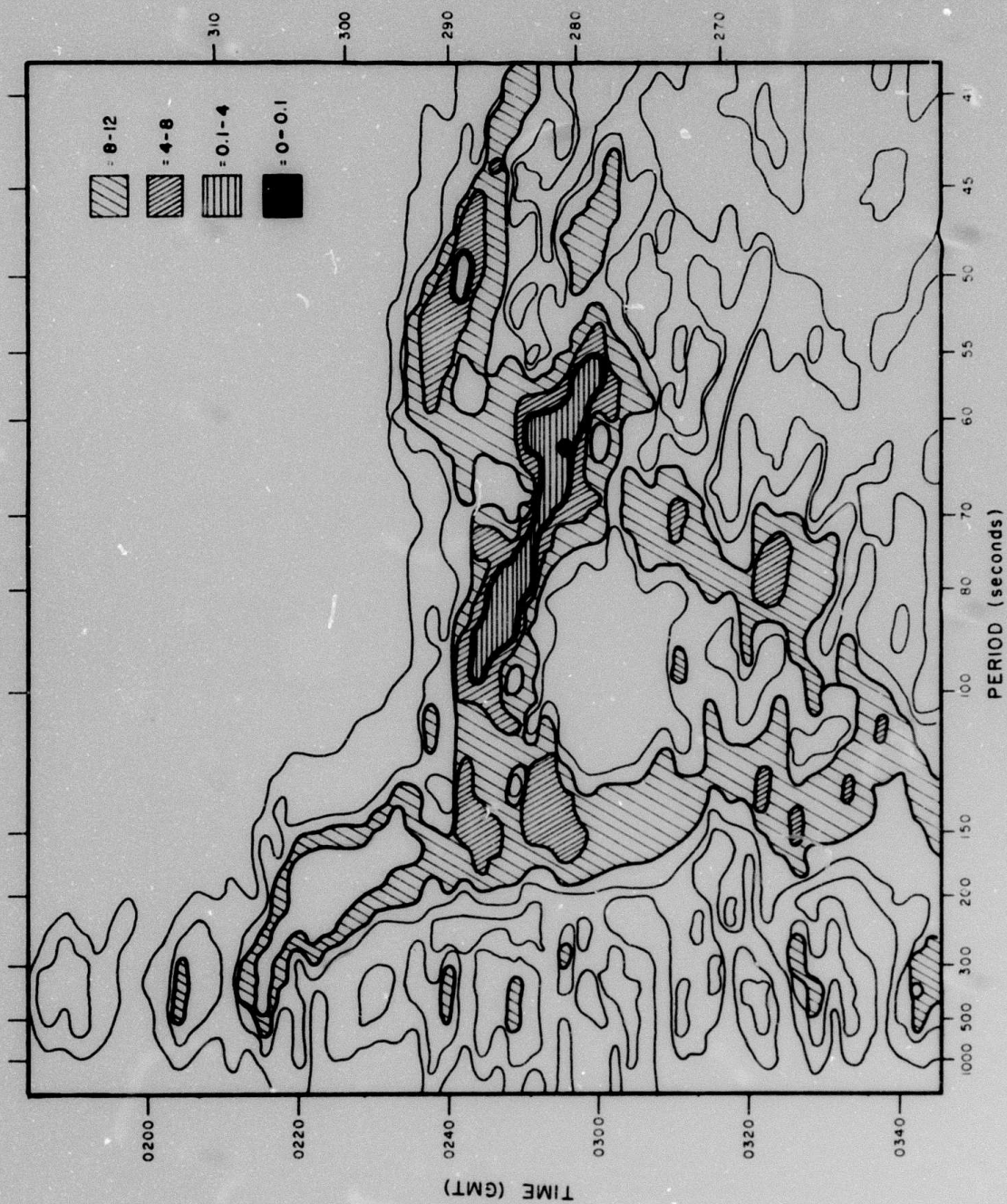


Figure 10. Time varying spectrum of the acoustic-gravity waves.

## CONCLUSIONS

1. Frequency-domain application of the least-squares estimation of signal propagation parameters can be a useful method for measuring phase velocity dispersion from array data. In the case of highly complicated dispersion curves, such as those observed for the Rayleigh waves studied here, multipath arrivals may be a significant cause of scatter.

2. The time-domain application is more accurate when the signal-to-noise ratio is high and the dispersion curves are relatively smooth. This procedure, when supplemented by recursively computed correlation functions, may be the more useful in signal detection.

3. The frequency-domain dispersion curves seem to be affected by the limits of the array response at the low-frequency end and the instrument system response at the high-frequency end. This might serve as a measure of array resolution in addition to the more standard methods.

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